



पाठ्यक्रम SYLLABUS

SCHEME OF EXAMINATION AND COURSES OF STUDY

FACULTY OF SCIENCE

M.Sc. MATHEMATICS

M.Sc. Mathematics (Previous) & (Final)

2009-10 से प्रभावी(w.e.f.)

सत्र 2013-14

महर्षि दयानन्द सरस्वती विश्वविद्यालय, अजमेर

NOTICE

1. Change in Statutes/Ordinances/Rules/Regulations/ Syllabus and Books may, from time to time, be made by amendment or remaking, and a candidate shall, except in so far as the University determines otherwise comply with any change that applies to years he has not completed at the time of change. The decision taken by the Academic Council shall be final.

सूचना

1. समय-समय पर संशोधन या पुनः निर्माण कर परिनियमों / अध्यादेशों / नियमों / विनियमों / पाठ्यक्रमों व पुस्तकों में परिवर्तन किया जा सकता है, तथा किसी भी परिवर्तन को छात्र को मानना होगा बशर्ते कि विश्वविद्यालय ने अन्यथा प्रकार से उनको छूट न दी हो और छात्र ने उस परिवर्तन के पूर्व वर्ष पाठ्यक्रम को पूरा न किया हो। विद्या परिषद द्वारा लिये गये निर्णय अन्तिम होंगे।

M.A/ M.SC. MATHEMATICS

There shall be 10 papers in all. Out of these five papers shall be offered in Previous and five in Final examination. Each paper shall be of 120 marks and of three hours duration.

Note : Each theory paper is divided into *three independent units*.

The question paper is divided into three parts Part-A, Part-B and Part-C.

Part A (24 marks) is *compulsory* and contains 10 questions (20 words each) and at least three questions from each unit, each question is of 2.4 marks.

Part B (24 marks) is *compulsory* and contains 4 questions at least one from each unit. Candidate is required to attempt all four questions. Each question is of 6 marks (50 words).

Part C (72 marks) contains 6 questions two from each unit. Candidate is required to attempt three questions taking one from each unit. Each question is of 24 marks (400 words).

M.A /M.SC. (PREVIOUS) EXAMINATION**SCHEME**

| Paper | Nomenclature | Teaching Hrs./week | Examination | |
|-------|---|-----------------------|------------------|--------------|
| | | | Duration Hrs. | Max Marks |
| I | Advanced Abstract Algebra | 6 | 3 | 120 |
| II | Real and Complex Analysis | 6 | 3 | 120 |
| III | Topology | 6 | 3 | 120 |
| IV | Differential Geometry and Tensors | 6 | 3 | 120 |
| V | Special Functions and Transform Calculus | 6 | 3 | 120 |
| | | | Total | 600 |

M.A /M.SC.(FINAL) EXAMINATION**SCHEME**

| Paper | Nomenclature | Teaching Hrs./week | Examination | |
|---|---|-----------------------|------------------|--------------|
| | | | Duration Hrs. | Max Marks |
| Compulsory : | | | | |
| VI | Functional Analysis and Integration Theory | 6 | 3 | 120 |
| VII | Advanced Differential and Integral Equations | 6 | 3 | 120 |
| Optionals : Any Three out of the following : | | | | |
| Opt. I | Generalised Hypergeometric Functions | 6 | 3 | 120 |
| Opt. II | Advanced Discrete Mathematics | 6 | 3 | 120 |
| Opt. III | Mathematical Modeling in Biology & Medicine | 6 | 3 | 120 |

| | | | | |
|-----------|----------------------------------|---|---|-----|
| Opt. IV | Continuum Mechanics | 6 | 3 | 120 |
| Opt. V | Fluid Dynamics | 6 | 3 | 120 |
| Opt. VI | Advanced Numerical Methods | 6 | 3 | 120 |
| Opt. VII | Relativity and Cosmology | 6 | 3 | 120 |
| Opt. VIII | Advanced Mathematical Statistics | 6 | 3 | 120 |
| Opt. IX | Operations Research | 6 | 3 | 120 |

M.A / M.SC. (PREVIOUS) - EXAMINATION**PAPER I - ADVANCED ABSTRACT ALGEBRA**

Duration : 3 Hrs.

Max. Marks : 120

Note : Each theory paper is divided into *three independent units*.

The question paper is divided into three parts Part-A, Part-B and Part-C.

Part A (24 marks) is *compulsory* and contains 10 questions (20 words each) and at least three questions from each unit, each question is of 2.4 marks.**Part B** (24 marks) is *compulsory* and contains 4 questions at least one from each unit. Candidate is required to attempt all four questions. Each question is of 6 marks (50 words).**Part C** (72 marks) contains 6 questions two from each unit. Candidate is required to attempt three questions taking one from each unit. Each question is of 24 marks (400 words).**Unit I**

Groups: Normal and subnormal series, composition series. Theorems on Isomorphism of groups, Class equation for finite group. Burnside theorem. Cauchy's theorem for finite abelian groups. Cauchy's theorem for finite groups, P-groups, Solvable Groups, Jordan-Holder Theorem, Nilpotent groups, Euclidean rings. Polynomial rings. Field theory-Extension fields, Algebraic and transcendental extensions, Separable and inseparable extensions, Normal extensions, Perfect fields, Finite fields. Primitive elements, Algebraically closed fields. Automorphisms of extensions. Galois extensions, Fundamental theorem of Galois theory.

Unit II

Solution of polynomial equations by radicals, insolvability of the general equation of degree 5 by radicals. Euclidean and polynomial rings, Polynomials over rational fields. The Eisenstein criterion, Polynomial rings over commutative ring, unique factorization domain, Chain conditions on rings.

Modules, sub modules, Quotient modules, Cyclic modules, Simple module. Semi Simple modules, Schuler's lemma. Free Modules.

Vector Spaces. Bases and co-ordinates Dimensions, Sylvester law of Nullity, Linear transformations and their representation as matrices, Change of basis, Dual space, dually paired vector spaces. Eigen values and Eigen vectors of a linear transformation.

Unit III

Diagonalisation, Bilinear, Quadratic and Hermitian forms, Inner product spaces Cauchy-Schwarz inequality, Orthogonal vectors. Orthogonal

complements, Orthonormal sets and bases.

Bessel's inequality for finite dimensional spaces, Gram-Schmidt orthogonalization process. Normal and self adjoint matrices and transformation, Unitary matrices and transformations, Principal axis theorem.

References:

- | | | |
|----------------------|------------------------|--------------------|
| 1. Algebra | : MacLane and Birkhoff | Macmillan Company. |
| 2. Topics in Algebra | : I.N. Herstein | Wiley Eastern Ltd. |
| 3. Linear Algebra | : S.Lang | Addison Wesley. |
| 4. Linear Algebra | : Hofmann and Kunz | Prentice Hall. |

PAPER II - REAL AND COMPLEX ANALYSIS

Duration: 3 Hrs

Max. Marks 120

Note : Each theory paper is divided into *three independent units*.

The question paper is divided into three parts Part-A, Part-B and Part-C.

Part A (24 marks) is *compulsory* and contains 10 questions (20 words each) and at least three questions from each unit, each question is of 2.4 marks.**Part B** (24 marks) is *compulsory* and contains 4 questions at least one from each unit. Candidate is required to attempt all four questions. Each question is of 6 marks (50 words).**Part C** (72 marks) contains 6 questions two from each unit. Candidate is required to attempt three questions taking one from each unit. Each question is of 24 marks (400 words).**Unit I (Measure Theory)**

Countable and non-countable sets. The Lebesgue measure of sets of real number, Measurable functions, Structure of Measurable functions, Weierstrass's Theorem on the approximation of continuous functions by polynomials, Lebesgue Integral of Measurable functions.

Summable functions, The space of square summable functions. Functions of finite variation. The Stieltjes integral. The indefinite Lebesgue integral.

Unit II

Complex integration, Cauchy's Goursat Theorem, Cauchy's integral formula, Higher order derivatives, Morera's theorem, Cauchy's inequality and Liouville's theorem. The fundamental theorem of algebra, Taylor's theorem, Maximum modulus principle, Schwarz lemma, Laurent's series, Isolated singularities, Meromorphic functions, The argument principle, Rouché's theorem, Inverse function theorem.

Unit III

Residues, Cauchy's residue theorem, Evaluation of integrals, Branches of many valued functions with special reference to $\arg z$, $\log z$ and z^a .

Spaces of analytic functions, Hurwitz's theorem. Montel's theorem, Riemann mapping theorem.

Weierstrass' factorization theorem, Gamma function and its properties, Riemann Zeta function. Riemann's functional equation. Runge's theorem, Mittag-Leffler's theorem, Analytic continuation. Uniqueness of direct analytic

continuation.

References:

1. Lebesgue Measure and Integration : P.K.Jain & V.P.Gupta
2. Methods of Real Analysis : Goldberg
3. Real Analysis : H.L.Royden
4. Theory of functions of Real Variable Vol. I. : I.P. Natanson
5. Complex Analysis : R.V.Churchil

PAPER III - TOPOLOGY

Duration: 3Hrs.

Max.Marks: 120

Note: Each theory paper is divided into *three independent units*.
The question paper is divided into three parts Part-A, Part-B and Part-C.

Part A (24 marks) is *compulsory* and contains 10 questions (20 words each) and at least three questions from each unit, each question is of 2.4 marks.

Part B (24 marks) is *compulsory* and contains 4 questions at least one from each unit. Candidate is required to attempt all four questions. Each question is of 6 marks (50 words).

Part C (72 marks) contains 6 questions two from each unit. Candidate is required to attempt three questions taking one from each unit. Each question is of 24 marks (400 words).

Unit I (Metric Spaces)

Metric spaces, Bounded and bounded metric spaces, Open and closed sets in a metric space, Cantor's ternary set, Closure Bases, Product spaces, Topological spaces, Sequence and subsequence in metric space, Cauchy sequence, Complete metric spaces, Completion of a metric space.

Unit II (Topology)

Topological spaces, Subspaces, Open sets, Closed sets, Neighbourhood system, Continuous mapping and Homeomorphism bases, and sub basis, Cauchy's sequences, Nets, Filters, Complete metric spaces, Product spaces, Quotient spaces.

Unit III

Compact and locally compact spaces, Tychonoff's one point compactifications, Separation axioms, Normal spaces, connected and locally connected spaces, Continuity and connectedness and compactness, Hausdorff spaces, Regular spaces.

References:

1. Topological Spaces. : Kowalsky
2. General Topology. : Kelly
3. Introduction to Topology and Modern Analysis : G.F. Simmons
4. Introduction to General Topology : K.D. Joshi
5. General Topology. : Gautam and Santi Naryan

PAPER IV - DIFFERENTIAL GEOMETRY AND TENSORS

Duration: 3Hrs.

Max.Marks: 120

Note: Each theory paper is divided into *three independent units*.
The question paper is divided into three parts Part-A, Part-B and Part-C.

Part A (24 marks) is *compulsory* and contains 10 questions (20 words each) and at least three questions from each unit, each question is of 2.4 marks.

Part B (24 marks) is *compulsory* and contains 4 questions at least one from each unit. Candidate is required to attempt all four questions. Each question is of 6 marks (50 words).

Part C (72 marks) contains 6 questions two from each unit. Candidate is required to attempt three questions taking one from each unit. Each question is of 24 marks (400 words).

Unit I

Curves in space (R^3): Space curves, Path, arc length, tangent line, contact of a curve and surface. Inflectional tangent, the osculating Plane. Tangent at any point of a surface $f(x,y,z) = 0$, Normal plane, Principal normal and Binormal, Curvature, Torsion and Skew curvature. Serret-Frenet formulae. Helices, Fundamental theorems for space curves, Circle of curvature, Osculating sphere.

Concept of surface and fundamental forms: Definition of surface, Regular point and singularities on a surface, Tangent plane and normal, First fundamental form, Relation between E, F, G and H. Second fundamental form, Weingarten equations, angle between parametric curves. Direction coefficients.

Unit II

Curves on a surface: Curvature of normal section, Meusnier theorem, Principal directions and Principal curvatures, Mean curvature, first curvature and total curvature, Minimal surface, Navel point, lines of curvature, Envelope, Edge of regression, Ruled surfaces, Developable surface, Asymptotic lines.

Transformation of Co-ordinates, Covariant, Contravariant and Mixed tensors, Invariants, Addition, Subtraction and Multiplication of Tensors, contraction of tensors, Quotient law of tensors, Fundamental tensors, Length of curve, Associated tensors.

Unit III

Christoffel symbols, Covariant differentiation of tensors, Law of covariant differentiation. Geodesics, Null Geodesics, Geodesics co-ordinates, Parallelism. Covariant derivative, Riemann Christoffel tensor, Curvature tensor, Ricci tensor, Bianchi identities, Riemann curvature. Flat space, Space of constant curvature.

References:

1. Differential Geometry : C.E. Weatherburn
2. Differential Geometry : H.C. Sinha

- | | | | |
|----|------------------|---|-----------------|
| 3. | Tensor Analysis | : | I.S.Sokolinkoff |
| 4. | Tensor Calculus | : | Berry Spain |
| 5. | Cartesian Tensor | : | A.M.Goodbody |

PAPER V - SPECIAL FUNCTIONS AND TRANSFORM CALCULUS

Duration : 3 Hrs. Max Marks: 120

Note : Each theory paper is divided into *three independent units*.
The question paper is divided into three parts Part-A, Part-B and Part-C.

Part A (24 marks) is *compulsory* and contains 10 questions (20 words each) and at least three questions from each unit, each question is of 2.4 marks.

Part B (24 marks) is *compulsory* and contains 4 questions at least one from each unit. Candidate is required to attempt all four questions. Each question is of 6 marks (50 words).

Part C (72 marks) contains 6 questions two from each unit. Candidate is required to attempt three questions taking one from each unit. Each question is of 24 marks (400 words).

Unit I

Hyper-Geometric Functions, Legendre's Polynomial, Associated Legendre's functions, Bessel's functions, Recurrence relations, Orthogonal Properties, Hermite and Laguerre Polynomials their generating functions and general integral properties.

Unit II

Laplace Transform: Definition and properties, Rules of manipulation : Laplace transform of derivatives : Inverse Laplace transform, Complex inversion formula, Theorems of Laplace transform.

Fourier Transform: Fourier sine and cosine transform. Convolution theorems. Fourier transform of derivative.

Hankel Transform: Definition and elementary properties : Inversion theorem, Hankel transform of derivatives, Parseval Theorem.

Unit III

Application to the solution of ordinary differential equations with constant coefficients and with variable coefficient, Simultaneous ordinary differential equations. Partial differential equations, Integral and difference equations, Application to the solution of boundary value problems. Application to the solution of partial differential equations.

References :

- | | | | |
|----|--|---|----------------------|
| 1. | Special function (chapters 6, 8, 10, 11, & 12) | : | Rainville E.D |
| 2. | The use of integral Transforms McGraw Hill | : | Sneddon I.N |
| 3. | Theory and problems of Laplace transform | : | Spegal M.R |
| 4. | Integral Transforms | : | Sharma and Vasishtha |

M.A. / M.SC.(FINAL) - EXAMINATION COMPULSORY PAPERS

PAPER VI - FUNCTIONAL ANALYSIS AND INTEGRATION THEORY

Duration : 3Hrs.

Max. Marks 120

Note : Each theory paper is divided into *three independent units*.
The question paper is divided into three parts Part-A, Part-B and Part-C.

Part A (24 marks) is *compulsory* and contains 10 questions (20 words each) and at least three questions from each unit, each question is of 2.4 marks.

Part B (24 marks) is *compulsory* and contains 4 questions at least one from each unit. Candidate is required to attempt all four questions. Each question is of 6 marks (50 words).

Part C (72 marks) contains 6 questions two from each unit. Candidate is required to attempt three questions taking one from each unit. Each question is of 24 marks (400 words).

Unit I (Functional Analysis)

Normed linear spaces, Banach Spaces and their examples, Continuous linear transformations. The open mapping theorem, Closed graph theorem, Uniform boundedness theorem, Continuous linear functionals, Hahn-Banach theorem.

Unit II

Inner product spaces, Hilbert spaces and their examples, Cauchy Schwarz's inequality, Parallelogram law, Orthogonal complements, Orthonormal sets, Bessel's inequality, Gram-Schmidt orthogonalization process, Riesz representation theorem, Operators and projections.

Unit III (Integration Theory)

Signed measure. Hahn decomposition theorem, mutually singular measures. Radon-Nikodym theorem. Lebesgue decomposition. Riesz representation theorem. Extension theorem (Caratheodory), Lebesgue-Stieltjes integral, product measures, Fubini's theorem, Differentiation and Integration. Decomposition into absolutely continuous and singular parts.

Baire sets, Baire measure, continuous functions with compact support. Regularity of measures on locally compact spaces. Integration of continuous function with compact support, Riesz-Markoff theorem.

References:

- | | | | |
|----|--|---|---|
| 1. | Introduction to Topology and Modern Analysis | : | GG Simmons: McGraw Hill Book company, Chapters 2, 9, and 10 (1963). |
| 2. | Elements of Functional Analysis | : | L. A. Luesternik and L. J Sobolev: Hindustan Publishing Company (1974). |
| 3. | Introduction to Functional Analysis | : | A.E. Taylor, John Wiley and |

4. Functional of Modern Analysis : sons. (1958)
J.Dieudonne (1969). Academic Press.
5. Fundamental analysis : Kosaku Yosida (1974)
6. Fundamental Analysis : B.Choudhary and Sudarshan
with Application Nanda Wiley-Eastern
Limited.(1989)
7. Applied Functional Analysis : A.V. Balakrishnan, Springer -
Verleg, New York.
8. Linear Operators Vol.I&II : N.Dunford, Interscience
1958,1963.

PAPER VII - ADVANCED DIFFERENTIAL AND INTEGRAL EQUATIONS

Duration : 3Hrs. Max. Marks: 120

Note : Each theory paper is divided into *three independent units*.
The question paper is divided into three parts Part-A, Part-B and Part-C.

Part A (24 marks) is *compulsory* and contains 10 questions (20 words each) and at least three questions from each unit, each question is of 2.4 marks.

Part B (24 marks) is *compulsory* and contains 4 questions at least one from each unit. Candidate is required to attempt all four questions. Each question is of 6 marks (50 words).

Part C (72 marks) contains 6 questions two from each unit. Candidate is required to attempt three questions taking one from each unit. Each question is of 24 marks (400 words).

Unit I (Differential Equations)

Existence and Uniqueness of solution $dy/dx = f(x,y)$. Green's function. Sturm- Liouville Boundary value problem. Cauchy problem and characteristics, Classification of Second order P.D.E. Separation of variables for heat equation. Wave equation and Laplace Equation.

Unit II (Calculus of Variations)

Linear functional, Minimal functional theorem, General variation of a function Euler-Lagrange's equation, Variational Methods for Boundary value problems in ordinary and partial differential equations.

Unit III (Integral Equations)

Linear Integral equation of the first and second kind of Fredholm and Volterra types, Solution by successive substitutions and successive approximations, Solution equation with separable kernels. The Fredholm alternative Hilbert Schmidt theory for symmetric kernels.

References :

1. Integral Equations, : Lovitte W.V Dover Publications.
2. Linear Integral Equations, : Kanwal R.P Academic Press New York.

OPTIONAL PAPERS (ANY THREE OF THE FOLLOWING) OPT. I - GENERALIZED HYPERGEOMETRIC FUNCTIONS

Duration: 3hrs.

Max Marks : 120

Note : Each theory paper is divided into *three independent units*.
The question paper is divided into three parts Part-A, Part-B and Part-C.

Part A (24 marks) is *compulsory* and contains 10 questions (20 words each) and at least three questions from each unit, each question is of 2.4 marks.

Part B (24 marks) is *compulsory* and contains 4 questions at least one from each unit. Candidate is required to attempt all four questions. Each question is of 6 marks (50 words).

Part C (72 marks) contains 6 questions two from each unit. Candidate is required to attempt three questions taking one from each unit. Each question is of 24 marks (400 words).

Unit I

Generalized Hypergeometric Functions : Definition, Convergence conditions for ' pFq ' differential equation and its solution, Watson's, Dixon's, Whipple's and Saalschutz theorems for the series ${}_3F_2$ with unit argument, Fundamental theorem due to thomae.

Unit II

Contour integral representation for pFq , Euler's type integrals involving pFq . Special cases. Product formulae due to Ramanujan, Preece and Bailey. Mac Roberts E-function, Definition and convergence conditions, Recurrence relations and Integrals involving E-functions.

Unit III

Meijer's G-Function : Definition, Nature and convergence conditions for the contours, special cases, Identities, Transformation formulas, differentiation formulas, recurrence relations, Contigues functions, relations. Simple finite and infinite integrals involving G-function, Mellin and Laplace transforms of G-function.

References :

1. Bailey, W. N., : Generalised Hypergeometric Series, Cambridge University Press, Cambridge, (1935)
2. Mathai A.M. and : Generalised Hypergeometric functions with applications in Saxena, R.K. Statistics and Physical Sciences, Lecture Note in Mathematics, 348 Springer verlag, New York, (1973). (Chapters 1 to 4).
3. Mathai A.M. and : The H-fuction with applications in Statistics and other disciplines, Wiley Eastern Ltd., New Delhi, (1978) (Chapters 1 to 3)
4. Raninville E.D. : Special functions, The MacMaillan Co., (1960)

OPT. II - ADVANCED DISCRETE MATHEMATICS

Duration : 3Hrs.

Max. Marks: 120

Note : Each theory paper is divided into *three independent units*.

The question paper is divided into three parts Part-A, Part-B and Part-C.

Part A (24 marks) is *compulsory* and contains 10 questions (20 words each) and at least three questions from each unit, each question is of 2.4 marks.**Part B** (24 marks) is *compulsory* and contains 4 questions at least one from each unit. Candidate is required to attempt all four questions. Each question is of 6 marks (50 words).**Part C** (72 marks) contains 6 questions two from each unit. Candidate is required to attempt three questions taking one from each unit. Each question is of 24 marks (400 words).**Unit I**

Formal logic-structures, Symbolic representation and tautologies, quantifiers, predicates, and validity, propositional logic. Semi groups and monoids, Definition and examples of semi groups and monoids, congruence relation and semi groups, and sub monoids, direct products, Basic homomorphism theorem.

Lattices: Lattices as partially ordered sets, their properties, lattices as algebraic system, sub lattices, direct products and homomorphism, some special lattices e.g. complete complemented and distributive lattices.

Unit II

Boolean Algebras: Boolean algebras as lattices. Various Boolean identities, the switching algebra example, Sub algebras, Direct product and homomorphism, join-irreducible elements, Atoms and minterms, Boolean forms and their equivalence, Minterm Boolean forms, Some of product canonical forms, Minimization of Boolean functions, Application of Boolean algebra to switching theory (using AND, OR & NOT Gates). The Karnaugh method.

Group theory: Definition of undirected graph, Paths, Circuits, Cycles and sub graphs, Induced sub graphs, Degree of vertex, Connectivity, Planner graphs and their property. Trees, Euler's formula for connected planner graphs, complete and complete bipartite graphs, Kuratowski's theorem (statement only) and its use, Spanning trees, Cut sets, Fundamental cut sets and cycles, Minimal spanning trees and Krushal's algorithm, Matrix representation of graphs, Eulers theorem on the existence of Eulerian path and circuits, Directed graphs, in degree and out degree of vertex, Weighted undirected graphs. Dijkstra's algorithm strong connectivity and Warshall's algorithm, Directed trees, Search trees, Tree traversals.

Unit III

Introductory Computability Theory: Finite state Machines and their transition. Table diagrams. Equivalence of finite state machine. Reduced machine, Homomorphism finite automata. Acceptors. Non-deterministic finite automata and equivalence of its power to that of deterministic finite automata.

Moore and Mealy machines. Turning machine and partial recursive functions. Grammars and Languages: Phase structures grammars, rewriting rules, derivations, essential forms, language generated by a grammar, regular context free, and context sensitive grammar and languages, regular sets and regular expressions and the pumping lemma, Kleene's theorem.

Notion of syntax analysis, polish notation, conversion of infix. Expression to polish notations. The reverse polish notation.

References:

1. Discrete mathematical structures with applications to computer science : J.P. Tremblay and R. Manohar McGraw Hill Book Co. 1997.
2. Finite Mathematics International edition, : Seymour Lipschitz McGraw Hill Book co. New York. 1983.
3. Elements of Discrete Mathematics, : C.Liu, McGraw Hill Book Co.
4. Graph theory with application to engineering and computer science, : N. Deo Prentice Hall of India.

OPT. III - MATHEMATICAL MODELING IN BIOLOGY AND MEDICINE

Duration : 3Hrs.

Max. Marks: 120.

Note : Each theory paper is divided into *three independent units*.

The question paper is divided into three parts Part-A, Part-B and Part-C.

Part A (24 marks) is *compulsory* and contains 10 questions (20 words each) and at least three questions from each unit, each question is of 2.4 marks.**Part B** (24 marks) is *compulsory* and contains 4 questions at least one from each unit. Candidate is required to attempt all four questions. Each question is of 6 marks (50 words).**Part C** (72 marks) contains 6 questions two from each unit. Candidate is required to attempt three questions taking one from each unit. Each question is of 24 marks (400 words).**Unit I**

Introduction - Microbial populations. Single-species. Non-Age structured. Population models, Age-structured population models. Two-species population models.

Unit II

Epidemic models, Models in genetics.

Unit III

Models for blood flows. Optimization models in biology and medicine.

References:

1. Mathematical Models in Biology and Medicine : J.N. Kapoor Affiliated East-West Press Private Limited New Delhi- Bangalore.

(Ch.1,2,3,4,5,8,9,11,&14).

OPT. IV - CONTINUUM MECHANICS

Duration: 3hrs.

Max Marks : 120

Note : Each theory paper is divided into *three independent units*.

The question paper is divided into three parts Part-A, Part-B and Part-C.

Part A (24 marks) is *compulsory* and contains 10 questions (20 words each) and at least three questions from each unit, each question is of 2.4 marks.**Part B** (24 marks) is *compulsory* and contains 4 questions at least one from each unit. Candidate is required to attempt all four questions. Each question is of 6 marks (50 words).**Part C** (72 marks) contains 6 questions two from each unit. Candidate is required to attempt three questions taking one from each unit. Each question is of 24 marks (400 words).**Unit I**

Cartesian Tensors. Index notations and transformation. Laws of Cartesian tensors. Additions. Subtraction and multiplication of Cartesian tensor. Gradient of a scalar function. Divergence of a vector function and curl of a vector function using the index notation. The identity Stokes Gauss and Green's theorems. The continuum approach classification of continuous media. Body forces and surface forces. Components of stress tensor. Forces and moment equation of equilibrium. The stress quadric. Principle stresses and Principle axes. Stress invariants and the stress deviator tensor. Maximum shearing stress.

Unit II

Lagrangian and Eulerian description of deformation of flow.

The comoving derivative. Velocity and acceleration, The continuity equation.

Strain tensors, The linear rotation tensor and rotation vector. Analysis of rotation displacement, Geometrical meaning of the components of the linear strain tensor, Principle axis theory for the linear strain tensor, properties of linear strain tensors. The linear cubical dilatation. Compatibility equations for the linear strain components. The rate of strain tensors and the velocity tensor, The rate of rotation vector and the vorticity. Properties of the rate of strain tensor, Rate of cubical dilatation, Law of conservation of mass and of motion, Kinetic equation of state, The first and the second law of thermodynamics and the dissipation function.

Unit III

Application: Linear elasticity : Assumption and basic equations, Generalised Hooke's Law for an isotropic Homogeneous solid, Compatibility equations. Classification of types of problems in linear elasticity, The principle of superposition, the Strain Energy function. The uniqueness theorem P.P. Relationship and the work kinetic energy equation, Irrotational flow and the velocity potential, Kinetic equation of state and the First Law of

Thermodynamics. The equation of continuity. The equations of motion, Vorticity-Stream surface for inviscid flow, Bernoulli's equations, Irrotational flow and the velocity potential, similarity parameters and fluid flow.

References:

- | | |
|---|---|
| 1. Continuum Mechanics, | : D Frederic and T.S.Chang |
| 2. Continuum Mechanics (Schaum Series) | : Mase. G.E |
| 3. Mechanics Deformable bodies | : Sommerfeld A |
| 4. An Introduction to Continuum Mechanics | : Morton E. Gurtin |
| 5. Mathematical Theory of Continuum Mechanics | : Rabindra Nath Chatterjee (Narosa Publishing House, Delhi) |

OPT. V - FLUID DYNAMICS

Duration : 3Hrs.

Max. Marks: 120

Note : Each theory paper is divided into *three independent units*.

The question paper is divided into three parts Part-A, Part-B and Part-C.

Part A (24 marks) is *compulsory* and contains 10 questions (20 words each) and at least three questions from each unit, each question is of 2.4 marks.**Part B** (24 marks) is *compulsory* and contains 4 questions at least one from each unit. Candidate is required to attempt all four questions. Each question is of 6 marks (50 words).**Part C** (72 marks) contains 6 questions two from each unit. Candidate is required to attempt three questions taking one from each unit. Each question is of 24 marks (400 words).**Unit I**

Kinematic of ideal fluid, Lagrange's and Euler's method, Equation of continuity in Cartesian, Polar and cylindrical co-ordinates, Boundary surfaces, stream lines, path line, velocity potential, Rotational and Irrotational motion, Equation of motion, Bernoulli's theorem, D'Alembert's paradox, Euler's momentum theorem, Helmholtz, Cauchy's integrals, Motion due to impulsive forces.

Unit II

Motion in two dimensions, Stream function, Irrotational motion, Complex potential, Sources, Sinks, Doublets and images, Motion of circular and elliptical cylinder, Motion of sphere, Viscosity, Analysis of stress, Relation between stress and rate of strain, dynamical similarity and inspection and dimensional analysis Buckingham theorem, Physical importance of non-dimensional parameters. Reynolds number, Froude number, Mach number, Prandtl number and Grashoff number, Navier-Stoke's equations. Some exact solutions of Navier-Stoke's equations, Plane Couette flow, Plane Poiseuille flow, Generalised plane Couette flow, Hagen-Poiseuille flow, Flow in tubes in uniform cross-section.

Unit III

Flow in convergent and divergent channels, stagnation point flows. Flow due to a rotating disc, flow due to a plane wall suddenly set in motion (Stokes first problem), Flow due to an oscillating plane wall (Stokes's second problem), stating Flow in a pipe, Theory of very slow motion, Stokes's flow past a sphere Oseen's flow past a sphere, Lubrication theory.

References:

- | | | |
|---------------------------------|---|-------------------|
| 1. A Text book on Hydrodynamics | : | M. Ray |
| 2. A Treatise on Hydrodynamics | : | Ramsay and Besant |
| 3. Viscous Fluid Dynamics | : | J.L. Bansal |
| 4. Fluid Dynamics | : | Shanti Swaroop |

OPT. VI - ADVANCED NUMERICAL METHODS

Duration : 3Hrs.

Max. Marks: 120

- Note 1.** The use of non programmable scientific calculator is permissible. A note to this effect be mentioned in the question paper also.
- Note 2.** Each theory paper is divided into *three independent units*. The question paper is divided into three parts Part-A, Part-B and Part-C.
- Part A** (24 marks) is *compulsory* and contains 10 questions (20 words each) and at least three questions from each unit, each question is of 2.4 marks.
- Part B** (24 marks) is *compulsory* and contains 4 questions at least one from each unit. Candidate is required to attempt all four questions. Each question is of 6 marks (50 words).
- Part C** (72 marks) contains 6 questions two from each unit. Candidate is required to attempt three questions taking one from each unit. Each question is of 24 marks (400 words).

Unit I

Iterative Methods: Simple iteration theory of iteration, Acceleration of convergence. Methods for multiple and complex roots. Newton-Raphson Method for simultaneous equations, Convergence of iteration process in the case of several unknown.

Solution of Polynomial Equations: Polynomial Evaluation, Real and complex roots. Synthetic division. The Birge-Vieta. Bairstow and Graeffe's root squaring methods.

Unit II

System of simultaneous equation (linear): Direct methods-Methods of determination. Gauss-elimination, Gauss-Jordan, Cholesky, Partition Methods of Successive, Approximate-Conjugate Gradient, Gauss and Jacobi iteration, Gauss seidel iteration and relaxation methods.

Eigen value Problems: Basic properties of Eigen values and Eigen Vector, Power method, Method for finding all Eigen pairs of a Matrix. Complex Eigen values.

Curve fitting and Function Approximation: Least square error

criterion Linear regression, Polynomial fitting and other curve fitting. approximation of functions by Taylor series and Chebyshev Polynomials.

Unit III

Numerical solution of Ordinary Differential Equations: Taylor Series method. Euler's and modified Euler's methods. Runge-kutta method upto fourth order.

Multistep method (Predictor-corrector strategies). Stability Analysis-Single and multistep methods.

Difference methods for BVPs ordinary Differential Equations Boundary value problems (BVP's) Shooting methods. Finite difference method. Difference scheme for non linear boundary value problems of the

Type $y' = f(x, y)$ and $y'' = f(x, y, y')$

References:

- | | | |
|---|---|------------------------|
| 1. Numerical Analysis | : | Jain, Iyenger and Jain |
| 2. Numerical Method | : | S.S. Sastry |
| 3. Numerical Solution of Differential equations | : | Jain M.K. |

OPT. VII - RELATIVITY AND COSMOLOGY

Duration : 3Hrs.

Max. Marks: 120

- Note:** Each theory paper is divided into *three independent units*. The question paper is divided into three parts Part-A, Part-B and Part-C.
- Part A** (24 marks) is *compulsory* and contains 10 questions (20 words each) and at least three questions from each unit, each question is of 2.4 marks.
- Part B** (24 marks) is *compulsory* and contains 4 questions at least one from each unit. Candidate is required to attempt all four questions. Each question is of 6 marks (50 words).
- Part C** (72 marks) contains 6 questions two from each unit. Candidate is required to attempt three questions taking one from each unit. Each question is of 24 marks (400 words).

Unit I

Bianchi identities and Einstein tensor, conformal curvature tensor, Algebraic classification of conformal curvature tensor, condition for flat space-time, Lorentz transformation. Mass-Energy formula. Minkowski's n dimensional continuum, space-like and time-like intervals.

Unit II

Principle of equivalence and Principle of general co-variance, Newtonian approximation of relativistic equations of motion. Einstein's field equations and its Newtonian approximation. Schwarzschild exterior solution and its isotropic form. Planetary orbits and analogues of Kepler's Laws in general relativity. Advance of perihelion of Mercury. Bending of light rays in a gravitational field. Gravitational red shift of spectral lines, Radar-Echo delay test.

Energy- Momentum tensor of perfect fluid. Schwazschild internal solution. Boundary condition's.

Unit III

Conservation of electric charge. Transformation formula for electric charge and electric current densities, Maxwell's equations in vacuo. Propagation of electric and magnetic intensities. Transformation of electric and magnetic intensities, Lorentz invariance of Maxwell's equations in tensor form. Energy-momentum tensor of electromagnetic field. Electromagnetism in general relativity. Reissner-Nordstrom solution.

Static cosmological models. Einstein universe. De-sitter universe. Properties of these universe. Comparison with actual universe.

References:

1. Weatherborn C.E : An introduction of riemannian Geometry and tensor calculus
: Cambridge Univ. Press
2. Eddington A.S. : The mathematical Theory of Relativity :
Cambridge Univ. Press
3. Narlikar J.V : General Relativity and cosmology : The Mac
Millan & Co. Ind. Ltd.
4. Alder R. Bazim M. : Introduction to general relativity : McGraw hill
Inc. Schiffer M.
5. New E.P. : Electromagnetism and relativity: Harper and
row New York
6. Tolman R.C : Relativity. Thermodynamics & cosmology. Oxford
University.

OPT. VIII - ADVANCED MATHEMATICAL STATISTICS

Duration: 3Hrs.

Max. Marks: 120

Note: Each theory paper is divided into *three independent units*.
The question paper is divided into three parts Part-A, Part-B and Part-C.

- Part A** (24 marks) is *compulsory* and contains 10 questions (20 words each) and at least three questions from each unit, each question is of 2.4 marks.
- Part B** (24 marks) is *compulsory* and contains 4 questions at least one from each unit. Candidate is required to attempt all four questions. Each question is of 6 marks (50 words).
- Part C** (72 marks) contains 6 questions two from each unit. Candidate is required to attempt three questions taking one from each unit. Each question is of 24 marks (400 words).

Unit I

Sample spaces, Combination of events. Statistical independence, Conditional probability, Bays theorem, Repeated trials. Random Variable, Distribution function. Probability, Probability function, Density function, Mathematical expectation, Generating function (mfg and pgf)

continuous probability distribution, characteristic function, Fourier's Inversion, Cheby-Shev, weak and Strong law of large numbers, Normal Hypergeometric, Rectangular, Negative, Binominal, Beta, Gamma and Cauchy's distribution.

Unit II

Association of attributes. Index number, Introduction, Price-relatives, Quantity relatives, Value relatives, Link and Chain relatives, Aggregate methods, Fisher's Ideal Index.

Elementary sampling theory, Distribution of means of sampling from Binomial, Cauchy, Rectangular and normal distribution. Distribution of second order moments in sampling from normal population. Exact distribution of χ^2 , t , z and F , Statistics in samples from a normal population, Their simple properties and applications.

Unit III

Test of significance of difference between two means and two standard deviations for large samples with modification for small samples and taken from normal population.

Analysis of variance, simple cases (one criteria and two criteria of classification) Elementary statistical Theory of estimation. Fisher's criteria for the best estimator. Consistent, Efficient and sufficient estimator. Method of Maximum likelihood estimators and other methods of estimation. Method of least square.

References:

- | | |
|--|--------------------|
| 1. Mathematical Theory of Statistics | : Kapur and Saxena |
| 2. A first course in Mathematical Statistics | : Weatherburn |
| 3. The Advanced Theory of Statistics | : M.G Kendall |
| 4. Introduction of Mathematical Probability | : Uspensky |

OPT. IX - OPERATIONS RESEARCH

Duration: 3Hrs.

Max. Marks: 120

Note: Each theory paper is divided into *three independent units*.
The question paper is divided into three parts Part-A, Part-B and Part-C.

- Part A** (24 marks) is *compulsory* and contains 10 questions (20 words each) and at least three questions from each unit, each question is of 2.4 marks.
- Part B** (24 marks) is *compulsory* and contains 4 questions at least one from each unit. Candidate is required to attempt all four questions. Each question is of 6 marks (50 words).
- Part C** (72 marks) contains 6 questions two from each unit. Candidate is required to attempt three questions taking one from each unit. Each question is of 24 marks (400 words).

Unit I

Operations Research and its Scope. Necessity of Operations Research in Industry.

Linear Programming-Simplex Method. Theory of the Simplex Method.
Duality and Sensitivity Analysis.

Other Algorithms for Linear Programming Dual simplex method,
Parametric Linear Programming, Upper Bound Technique, Interior Point
Algorithm, Linear Goal Programming.

Transportation and Assignment Problems.

Unit II

Network Analysis - Shortest Path Problem, Minimum Spanning Tree
Problem, Maximum Flow Problem, Minimum cost flow problem, Network simplex
method, Project planning and control with PERT-CPM.

Dynamic Programming - Deterministic and Probabilistic Dynamic
programming.

Game theory - Two person, Zero-sum games, Games with mixed
strategies, Graphical Solution, Solution by Linear Programming.

Integer Programming - Branch and Bound Technique.

Unit III

Application to Industrial problems - Optimal product mix and activity
levels. Petroleum refinery operations. Blending problems, Economic
interpretation of dual linear programming problems, Input-Output analysis,
Leontief system. Indecomposable and Decomposable economies.

Nonlinear Programming - One and Multi variable unconstrained
optimization. Kuhn-Tucker conditions for constrained optimization, Quadratic
programming. Separable programming, Convex programming, Non-convex
programming.

References :

1. F.S. Hiller and G.J. Lieberman, Introduction to Operation Research (Sixth edition), McGraw Hill International edition, Industrial engineering Series, 1995.
2. G Hadley, Linear Programming, Narosa Publishing House, 1995.
3. G Hadley, Nonlinear and Dynamic Programming, Addison-Wesley, Reading Mass.
4. Kanti Swarup, P.K. Gupta and Man Mohan, Operations Research, Sultan Chand & Sons, New Delhi.
5. S.S. Rao, Optimization Theory and Applications, Wiley Eastern Ltd. New Delhi.
6. Prem Kumar Gupta and D.S. Hira, Operations Research - An Introduction S. Chand & Company Ltd., New Delhi.
7. N.S. Kambo, Mathematical Programming Techniques, Affiliated East-West Press Pvt. Ltd., New Delhi, Madras.